Kinematics Solution of the RV-3SB Robot Using Successive Screws

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***Abstract*—In this paper a complete kinematics analysis of the RV-3SB Mitsubishi robot is perform. An alternative methodology to address the kinematic problem based on the Screws theory is presented. Then, the design of a simulator to validate the methodology proposed, by comparing the values of the simulator with the real values of the robot is presented, MATLAB as a testing tool and programming language is used.**

***Index Terms*—Forward kinematics, Inverse kinematics, Screw Theory, Successive Screws, Simulator, MATLAB.**

I. Introduction

An industrial robot is an electromechanical device which is used for industrial automation tasks that included but are not limited to, assembly, painting, palletizing, manufacturing, pick and place, etc. A robot includes rigid bars, joints, generally one degree of freedom, and a power and electronic system.

In recent years the applications of Industrial robotics continue to increase, the major industries use robots are the in the automotive industry followed by the electronics industry. However, with the arrival of the fourth industrial revolution and the emergence of the new paradigm of collaborative robots, robotics is having a new boom while changing the way of interaction between human and robot.

For this reason it is necessary to know in detail the kinematic relationships that exist between the joints of the robot and the end effector, this will allow us to know the location of its end effector, knowing the value of its joint variables, this is know as forward kinematic analysis. On the other hand, it is important to know the value that the joints take for a specific end effector trajectory, in this case the analysis is known as inverse kinematics [1], [2], [3].

Traditional methods for solving the kinematic problem consist of assigning reference frames to the joints and then applying the Denavit-Hartenberg method [4], [5], [6], however, this method is very susceptible to errors, especially when reference frames are assigned, it is also a relatively slow method. On the other hand, we have the method of successive screws [7], [8], a simpler and systematic method and also faster to apply, this method is based on identifying a single axis of rotation or translation called the screw axis and then applying the Rodrigues formula for the general displacement of a Rigid body [9]. As explained in later sections, the method is highly efficient and much easier.

This work is organized as follows, first, basic concepts are explained and the transformation matrix are obtained through Rodrigues's formula, then, successive screws is applied to solve the forward kinematics of an industrial robot. Moreover, the inverse kinematics is solved using kinematic decoupling and the matrices manipulation.

Finally, to validate the method proposed, real values were assigned to the robot using the 'Teach Pendant', and then compared with the values obtained through the equations and the absolute error were evaluated.

II. The Transformation Matrix and Screws Parameters

The general movement of a body and be defined by combined movement of a rotation and a translation around an unique axis, this axis is known as screw axis, and the motion is called Twist or screw displacement [10], [11], [12].

Although we can express a screw displacement using a vector form, the matrix form is recommended because it is easier to manipulate mathematically. So, lets considered the Fig. 1, in which we have a point P_1 expressed respect to fixed frame $OXYZ$, where $s = \left[s_x, s_y, s_z\right]^T$ is a unit vector along the screw axis direction and $s_0 = \left[s_{0x}, s_{0y}, s_{0z}\right]^T$ is a position vector that locate the screw axis.

The goal is to find an expression that related the location of the final point P_2 after applying a rotation of θ about the screw axis, followed by a translation of a t distance in parallel direction to the screw axis.

Thus, we can determine a matrix that relates the rotation and translation of a point P_1 resulting in the new point P_2.

Considering the rotation of P_1 to P_2^r, the new position of P, knowing the screw axis and the rotation angle, can be determined by,

$$r_2 = r_1 cos\theta + s \times r_1 sen\theta + s(r_1^T s)(1 - cos\theta) \qquad (1)$$

Eq. 1 is know as Rodrigues's formula for a spherical displacement, in the case of the translation, and considering Fig. 1 we note that

$$r_1 = p_1 - s_0, \qquad (2)$$

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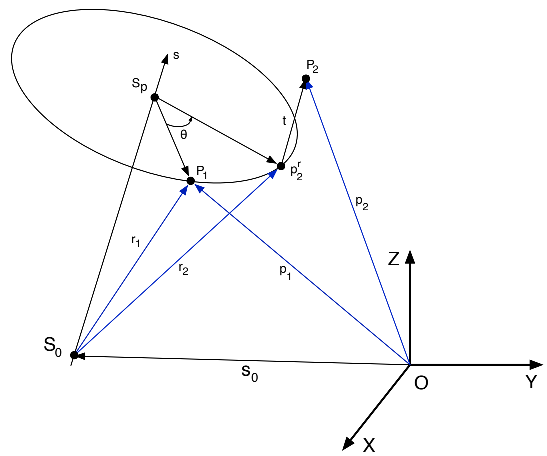

Fig. 1: Vector diagram of a spatial displacement [11].

$$r_2 = p_2 - s_0 - ts \tag{3}$$

Thus, substituting (2) and (3) into (1), and solving for p_2, we obtain

$$\begin{aligned} p_2 = s_0 + ts + (p_1 - s_0)cos\theta + s \times (p_1 - s_0)sen\theta + \\ s(\left[p_1 - s_0\right]^T s)(1 - cos\theta) \end{aligned} \tag{4}$$

In which (4) is known as Rodrigues's formula for a general displacement of a rigid body. Then, rewriting (4) in matrix form we have

$$p_2 = \left[Icos\theta + \tilde{s}sen\theta + s's(1 - cos\theta)\right](p_1 - s_0) + s_0 + ts \tag{5}$$

where,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tilde{s} = \begin{bmatrix} 0 & -s_z & s_y \\ s_z & 0 & -s_x \\ -s_y & s_x & 0 \end{bmatrix}, s = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$$

On the other hand, in (5), $\left[Icos\theta + \tilde{s}sen\theta + s's(1 - cos\theta)\right]$ is the rotation matrix. Then, substituting p_1 by ${}^Bp$ and p_2 by ${}^Ap$, yields

$${}^Ap = {}^AR_B \, {}^Bp + {}^Aq \tag{6}$$

Finally, (6) can be expressed as a homogeneous transformation matrix, thus

$${}^Ap = T \, {}^Bp \tag{7}$$

where T is a transformation matrix that relates P_1 with P_2, and the elements of T are,

$$\begin{aligned}
a_{11} &= (s_x^2 - 1)(1 - cos\theta) + 1, \\
a_{12} &= s_x s_y(1 - cos\theta) - s_z sen\theta, \\
a_{13} &= s_x s_z(1 - cos\theta) + s_y sen\theta, \\
a_{14} &= ts_x - s_{0x}(a_{11} - 1) - s_{0y}a_{12} - s_{0z}a_{13}, \\
a_{21} &= s_y s_x(1 - cos\theta) + s_z sen\theta, \\
a_{22} &= (s_y^2 - 1)(1 - cos\theta) + 1, \\
a_{23} &= s_y s_z(1 - cos\theta) - s_x sen\theta, \\
a_{24} &= ts_y - s_{0x}a_{21} - s_{0y}(a_{22} - 1) - s_{0z}a_{23}, \\
a_{31} &= s_z s_x(1 - cos\theta) - s_y sen\theta, \\
a_{32} &= s_z s_y(1 - cos\theta) + s_x sen\theta, \\
a_{33} &= (s_z^2 - 1)(1 - cos\theta) + 1, \\
a_{34} &= ts_z - s_{0x}a_{31} - s_{0y}a_{32} - s_{0z}(a_{33} - 1), \\
a_{41} &= 0, \; a_{42} = 0, \; a_{43} = 0, \; a_{44} = 1
\end{aligned} \tag{8}$$

III. Successive Screws

To apply the successive screw method we need to find a transformation matrix for each joint using the Rodrigues's formula, moreover, we need to know the vector position of the screw axis, and the rotation angle (θ) if we have a revolute joint or the translation distance (t) if we have a prismatic joint [13]. Next, we have to multiply each transformation matrix to obtain an overall matrix that relates the end-effector location with the system base (base of the robot).

In order to find this overall matrix, lets consider Fig. 2, the position of the robot can be chosen arbitrarily, however, it is recommended to choose the home position of the robot given by the manufacturer (position in which all joint values are zero) to obtain the same results that the real data.

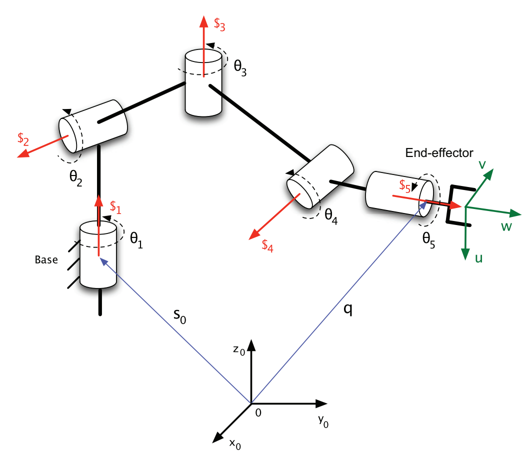

Fig. 2: Screws axes of a serial robot.

On the other hand, if we consider that the robot moves from a given location to a new one, the resulting location will be given by the premultiplication of the n joint's transformation matrices (successive screws), that is

$$T = T_1 \; T_2 \; T_3 \; T_4 \; T_5 \tag{9}$$

In general, to solve the direct kinematics we need the multiply the n transformation matrix corresponding to the n-th joint,

$$T = T_1\ T_2\ T_3\ \dots T_n \tag{10}$$

On the other hand, for the inverse kinematics, the goal is to find the joint's position to move the end-effector to a desire location, thus, the left term of (10) will be given.

IV. The RV-3SB Mitsubishi Robot

The RV-3SB robot is an industrial serial robot, has 6 degrees of freedom and is composed of revolute joints of one degree of freedom, as shown in Fig. 3.

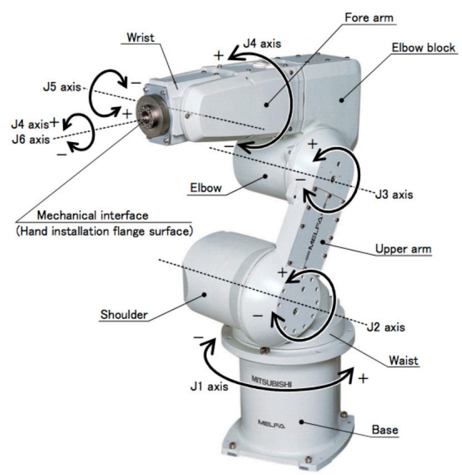

Fig. 3: The RV-3SB Mitsubishi Serial Robot.

Some of the main features of the RV-3SB robot are:

- 6 Degrees of Freedom
- Repeatability: ± 0.02 mm
- Maximum speed: 5500 mm/s
- Range of Motion (degrees): J1=340, J2=225, J3=191, J4=320, J5=240, J6=720
- Maximum speed in each joint (deg/seg): J1=250, J2=187, J3=250, J4=412, J5=412, J6=660
- Weight: 37 Kg

A. Forward Kinematics

The problem of the forward kinematic solution is to find the vector position (x, y, z) and the orientation $(roll, pitch, yaw)$ of the end-effector, given the state of the actuators $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$.

First, we have to chose a reference position, as mention before the reference position is arbitrary but is recommended to chose the home position given by the manufacturer.

Thus, lets to consider the position depicted in Fig. 4 as reference position. The reference position was chosen according to home robot position given by the manufacturer.

The analysis begins by identifying the rotation axis of each joint (screw axis), and then locating the vector position of the screw axis. The first, fourth and sixth joint axes, $\$_1$, $\$_4$ y $\$_6$,

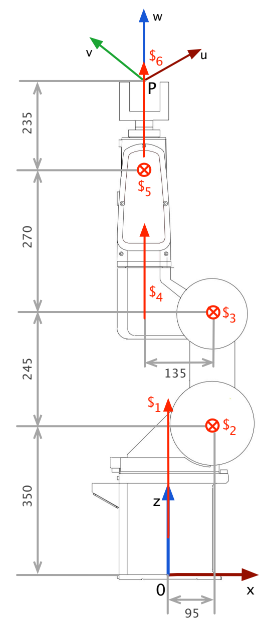

Fig. 4: Reference Position for the RV-3SB Mitsubishi Robot (Dimensions are in mm).

are all pointing up vertically in the positive z-direction; and the second, third, and fifth joint axes, $\$_2$, $\$_3$ y $\$_5$, are all pointing into the paper, in the positive y-direction.

The reference position of the end-effector is rotated 45° with respect to the system base, thus, will be given by

$$\begin{aligned} u &= [\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0]^T, v = [\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0]^T, \\ w &= [0, 0, 1]^T, p_0 = [-40, 0, 1100]^T \end{aligned} \tag{11}$$

At this reference position, the location of the screws axes with respect to the system base are shown in Table I.

TABLE I: Screw Axis Location of the RV-3SB Robot

Joint i	Screw Axis (S_i)	Screw Axis Location (S_{0i})
J_1	$[0, 0, 1]$	$[0, 0, 0]$
J_2	$[0, 1, 0]$	$[95, 0, 350]$
J_3	$[0, 1, 0]$	$[95, 0, 595]$
J_4	$[0, 0, 1]$	$[-40, 0, 0]$
J_5	$[0, 1, 0]$	$[-40, 0, 865]$
J_6	$[0, 0, 1]$	$[-40, 0, 0]$

Substituting the coordinates of the joint axes into (8), we obtain the transformation matrices $T_1, T_2, T_3, T_4, T_5, T_6$, thus, the overall transformation matrix is,

$$T = T_1\ T_2\ T_3\ T_4\ T_5\ T_6 \tag{12}$$

Therefore, the transformation of the point P is given by

$$p = Tp_0 \tag{13}$$

where $p_0 = [-40, 0, 1100, 1]^T$. The orientation of the end effector is,

$$R = T(1:3, 1:3)Rot(z, 45°) \tag{14}$$

where,

$$R = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \begin{bmatrix} cos45° & -sen45° & 0 \\ sen45° & cos45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

Note, we have multiplied by a rotation matrix resulting from rotate $45°$ about the z axis, $Rot(z, 45°)$, since the reference position of the end-effector. Finally, considering the elements of (20) as

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{16}$$

The Euler's Angles (Roll(ψ), Pitch(θ), Yaw(ϕ) of the end-effetor are computed as

$$\begin{aligned} \theta &= sen^{-1}(-r_{31}) \\ \psi &= atan2(r_{32}/c\theta, r_{33}/c\theta) \\ \phi &= atan2(r_{21}/c\theta, r_{11}/c\theta) \end{aligned} \tag{17}$$

B. Inverse Kinematics

The problem of the Inverse Kinematics is to find the states of the actuators $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$, given the position (x, y, z) and orientation $(roll, pitch, yaw)$ of the end-effector.

Since the last three joint axes intersect one to another in the fifth joint, the position and orientation can be decoupled. Thus, the position of the wrist center point ($P_{\$_5}$) of the fifth joint only depends on the first three joint variables, next, we solve the last three joint variables in terms of end-effector orientation.

Moreover, the target position of the wrist center point $P_{\$_5}$ is given by

$$P_{\$_5} = P - l_4 w \tag{18}$$

where l_4 is the distance between $P_{\$_5}$ and P in w direction.

In the other hand, the wrist center point is related with the fixed reference frame by

$$P_{\$5} = T_1 T_2 T_3 T_4 T_5 P_{\$5_0} \tag{19}$$

Multiplying both sides of (19) by T_1^{-1}, we obtain

$$T_1^{-1} P_{\$5} = T_2 T_3 P_{\$5_0} \tag{20}$$

Substituting the transformation matrices (T_1 to T_6) into (20) yields

$$\begin{bmatrix} p_x c\theta_1 + p_y s\theta_1 \\ p_y c\theta_1 - p_x s\theta_1 \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} 270 s\theta_{23} - 135 c\theta_{23} + 245 s\theta_2 + 95 \\ 0 \\ 270 c\theta_{23} + 135 s\theta_{23} + 245 c\theta_2 + 350 \\ 1 \end{bmatrix} \tag{21}$$

where, $\theta_{23} = \theta_2 + \theta_3$.

Equating terms in (21) we have:

$$p_x c\theta_1 + p_y s\theta_1 = 270 s\theta_{23} - 135 c\theta_{23} + 245 s\theta_2 + 95 \tag{22}$$

$$p_y c\theta_1 - p_x s\theta_1 = 0 \tag{23}$$

$$p_z = 270 c\theta_{23} + 135 s\theta_{23} + 245 c\theta_2 + 350 \tag{24}$$

From (23) two solutions of θ_1 are found immediately:

$$\theta_1 = tan^{-1} \frac{p_y}{p_x} \tag{25}$$

That is, $\theta_1 = \theta_1^*$ is a solution, $\theta_1 = \theta_1^* + \pi$ is also a solution, where $0 \leqslant \theta_1^* \leqslant \pi$. Note that only the first solution is feasible due to mechanical constrains.

Next, rewriting (22) and (24) as

$$K_1 - 245 s\theta_2 = 270 s\theta_{23} - 135 c\theta_{23} \tag{26}$$

$$K_2 - 245 c\theta_2 = 270 c\theta_{23} + 135 s\theta_{23} \tag{27}$$

where: $K_1 = p_x c\theta_1 + p_y s\theta_1 - 95$ y $K_2 = p_z - 350$

Summing the square of (26) and (27), and simplifying we have

$$e_1 s\theta_2 + e_2 c\theta_2 + e_3 = 0 \tag{28}$$

where:
$e_1 = 490K_1$,
$e_2 = 490K_2$, y
$e_3 = (270)^2 + (135)^2 - (245)^2 - K_1^2 - K_2^2$

Then, substituting into (28) the following identities

$s\theta_2 = \frac{2t_2}{1+t_2^2}$, y $c\theta_2 = \frac{1-t_2^2}{1+t_2^2}$, where $t_2 = tan(\frac{\theta_2}{2})$

yields

$$(e_3 - e_2)t_2^2 + 2e_1 t_2 + (e_3 + e_2) = 0 \tag{29}$$

Hence, solving (29) for t_2 we obtain

$$\theta_2 = 2tan^{-1} \frac{-e_1 \pm \sqrt{e_1^2 + e_2^2 - e_3^2}}{e_3 - e_2} \tag{30}$$

Therefore, we have two solutions for θ_2. Once θ_2 is known, (27) can be written as

$$f_1 s\theta_3 + f_2 c\theta_3 + f_3 = 0 \tag{31}$$

where:
$f_1 = -270 s\theta_2 + 135 c\theta_2$,
$f_2 = 270 c\theta_2 + 135 s\theta_2$, y
$f_3 = 245 c\theta_2 - K_2$

Substituting into (31) the following identities

$s\theta_3 = \frac{2t_3}{1+t_3^2}$, y $c\theta_3 = \frac{1-t_3^2}{1+t_3^2}$, where $t_3 = tan(\frac{\theta_3}{2})$

yields

$$(f_3 - f_2)t_3^2 + 2f_1 t_3 + (f_3 + f_2) = 0 \tag{32}$$

and solving for t_3, we obtain

$$\theta_3 = 2tan^{-1}\frac{-f_1 \pm \sqrt{f_1^2+f_2^2-f_3^2}}{f_3-f_2} \quad (33)$$

We have also two solutions for θ_3. In general there are at most six upper arm configurations corresponding to each given end-effector location.

As we mentioned before, the end-effector orientation depends only on the last three joint variables, that is only on the the rotational part of the transformation matrices T_i, thus

$$R_4R_5R_6 = R(z,\theta_4)R(y,\theta_5)R(z,\theta_6) \quad (34)$$

where,

$$R_4R_5R_6 = \begin{bmatrix} c\theta_4c\theta_5c\theta_6 - s\theta_4s\theta_6 & -c\theta_6s\theta_4 - c\theta_4c\theta_5s\theta_6 & c\theta_4s\theta_5 \\ c\theta_4s\theta_5 + c\theta_5c\theta_6s\theta_4 & -c\theta_4c\theta_6 - c\theta_5s\theta_4s\theta_6 & s\theta_4s\theta_5 \\ -c\theta_6s\theta_5 & s\theta_5s\theta_6 & c\theta_5 \end{bmatrix} \quad (35)$$

Furthermore, the desire orientation is given by

$$R^* = R_1R_2R_3R_4R_5R_6 = \begin{bmatrix} n & o & a \end{bmatrix} \quad (36)$$

Multiplying both sides of (36) by $(R_1R_2R_3)^{-1}$, we have

$$R_3^TR_2^TR_1^TR^* = R_4R_5R_6 \quad (37)$$

For convenience we define $\Delta = R_3^TR_2^TR_1^TR^*$, then, (37) can be written as

$$\Delta = R_4R_5R_6 \quad (38)$$

Expanding (38) yields

$$\Delta = \begin{bmatrix} c\theta_4c\theta_5c\theta_6 - s\theta_4s\theta_6 & -c\theta_6s\theta_4 - c\theta_4c\theta_5s\theta_6 & c\theta_4s\theta_5 \\ c\theta_4s\theta_5 + c\theta_5c\theta_6s\theta_4 & -c\theta_4c\theta_6 - c\theta_5s\theta_4s\theta_6 & s\theta_4s\theta_5 \\ -c\theta_6s\theta_5 & s\theta_5s\theta_6 & c\theta_5 \end{bmatrix} \quad (39)$$

Hence, from $\Delta(3,3)$ we have:

$$c\theta_5 = \delta_{33} \quad (40)$$

Since $s\theta_5 = \pm\sqrt{1-c^2\theta_5}$, where $c\theta_5 = \delta_{33}$, we obtain:

$$\theta_5 = tan^{-1}\frac{\pm\sqrt{1-c^2\theta_5}}{c\theta_5} \quad (41)$$

Eq. (41) yields two real solutions of θ_5 if $|\delta_{33}| < 1$. When $|\delta_{33}| = 1, \theta_5 = 0$ or π, for this case the forth and sixth joint are aligned. Moreover, the condition $|\delta_{33}| > 1$, can not occur physically.

Next, from elements $\Delta(1,3)$ and $\Delta(2,3)$, we have

$$\delta_{13} = c\theta_4s\theta_5 \quad (42)$$

$$\delta_{23} = s\theta_4s\theta_5 \quad (43)$$

Hence

$$\theta_4 = Atan2(\delta_{23}/s\theta_5, \delta_{13}/s\theta_5) \quad (44)$$

Finally, we solve θ_6 from the elements $\Delta(3,1)$ and $\Delta(3,2)$, that is

$$\delta_{31} = -c\theta_6s\theta_5 \quad (45)$$

$$\delta_{32} = s\theta_5s\theta_6 \quad (46)$$

Thus

$$\theta_6 = Atan2(\delta_{32}/s\theta_5, -\delta_{31}/s\theta_5) \quad (47)$$

Since the reference system of the end-effector is rotated 45°, respect to the fixed reference frame, the matrix R^* used to find Δ matrix must include this rotation.

From the end-effector orientation, given by the Euler's angles, the rotation matrix is

$$R = \begin{bmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi s\psi \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix} \quad (48)$$

Thus, R^* matrix is given by

$$R^* = R[Rot(z, 45°)]^T \quad (49)$$

We conclude that corresponding to a given position and orientation of the end-effector, we have up to ten inverse kinematics solutions.

V. Test Performed

In order to validate the equations obtained, a 5 sets of random values (Table II) for the joints were chosen and assigned to the robot using the Teach Pendant, then, the location of the end-effector (position and orientation) were registered. Next, the same sets of joint values were assigned to the kinematics equations. Table III and IV shown the obtained results for both position and orientation of the end-effector.

TABLE II: Random Joint values assigned

Trial	J_1	J_2	J_3	J_4	J_5	J_6
1	0	0	0	0	0	0
2	-13.94	-6.19	121.15	3.89	67.3	59.75
3	-43.75	-38.04	137.82	-1.61	86.29	-44.28
4	0.8	15.35	83.7	36.39	3.57	10.42
5	59.14	60.97	29.13	-33.31	-45.46	168.07

TABLE III: End-effector Position

	x		y		z	
Trial	x_{real}	x_{sim}	y_{real}	y_{sim}	z_{real}	z_{sim}
1	-40.01	-40	0	0	1100	1100
2	354.18	354.18	-72.73	-72.75	367.67	367.66
3	145.84	145.84	-148.74	-148.73	396.55	396.54
4	677.32	677.31	18.1	18.14	628.58	628.58
5	302.96	302.94	686.3	686.3	743.09	743.11

The results shown that the difference between the real values and those obtained by the kinematics equations are very small. Fig. 5 and Fig. 6 shown the position and orientation error. The Absolute Error (AE) for position was 0.011 mm, whereas for orientation AE was zero for $roll(\psi)$, $pitch(\theta)$ and 0.0063 degrees for $yaw(\phi)$ angle.

TABLE IV: End-effector Orientation

Trial	Roll (ψ) ψ_{real}	Roll (ψ) ψ_{sim}	Pitch (θ) θ_{real}	Pitch (θ) θ_{sim}	Yaw(ϕ) ϕ_{real}	Yaw(ϕ) ϕ_{sim}
1	0	0	0	0	45.01	45
2	-176.87	-176.87	-2.82	-2.82	59.67	59.66
3	-178.33	-178.33	-6.05	-6.05	135.63	135.63
4	101.92	101.92	-2.16	-2.16	92.5	92.5
5	13.22	13.22	-52.37	-52.37	-108.26	-108.26

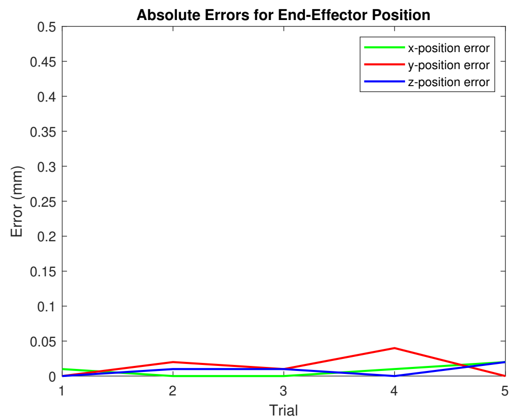

Fig. 5: Absolute error for end-effector position

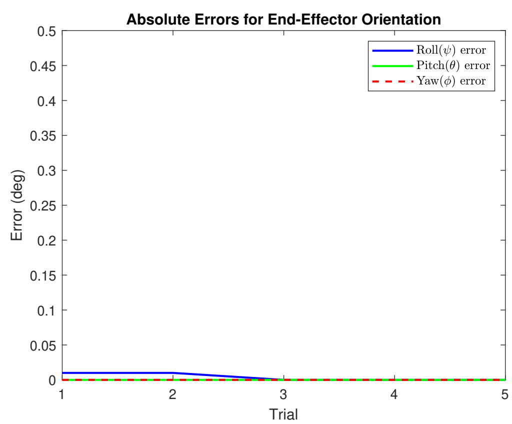

Fig. 6: Absolute error for end-effector orientation

VI. Discussion and Conclusions

In this paper the solution of the kinematics of serial robots using successive Screws was presented. The screw parameters and the transformation matrices based on the Rodrigues's formula were defined, the RV-3SB Mitsubishi robot was taken for testing.

A reference position of the robot was considered, next, the joint axes and screw axes location were determined, then, the transformation matrices were calculated, and forward and inverse kinematics solution was found. The reference position for the analysis can be taken arbitrarily, however, it is convenient to choose a position that easily allows us to determine the geometry of the robot and the joint axes location, in this case, the reference position was taken based on the *home position* of the robot, reference given by the manufacturer.

Although the most widely used method for solving serial robot kinematics is the Denavit and Hartenberg's Method, it is easy to make errors when the reference systems are chosen. In addition, applying the algorithm that the authors propose is not always simple and sometimes confusing.

On the other hand, the method of the successive screws is simpler to apply, first the joint axes and the screw axis location is identified, which is measured from the fixed reference frame (base system) to any point of the screw axis, and then the Rodrigues's formula is applied to determine the transformation matrices.

Finally, five sets of random values for joints were chosen and assigned to the robot using the Teach Pendant, then, the position and orientation of the end-effector were registered and compared with the results obtained from the equations. In general, the results shown very low values of Absolute Error (AE), indeed, practically identical to real values.

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