A new Approach for the Forward Kinematics of General Stewart-Gough Platforms

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***Abstract*—In this paper a complete forward kinematics analysis of a general Stewart-Gough platform is performed. A new methodology to deal with the forward kinematics problem based on a multibody formulation is presented. An iterative algorithm is proposed and the forward kinematics using a numerical method is solved. Finally, a design of a simulator using MATLAB as programming tool is presented.**

***Index Terms*—Forward Kinematics, MATLAB®, Multibody Formulation, Numeric Method, Parallel Platform, Quaternions.**

I. Introduction

A Parallel Robot is a type of mechanism that has multiple kinematic chains (limbs) connecting their base with a moving platform (called end effector). Normally, each kinematic chain has a series of links connected by joints. Because of this, parallel robots have many advantages compared to serial robots, such as high speed, low inertia, high stiffness and large payload capacity.

The kinematic analysis of a robot studies the relationship between the location and orientation of the end effector and its joints values. Because we could have as unknown actuators position given the end-effector position or vice versa, kinematics is classified in forward and inverse kinematics.

The solution of the forward kinematics consists of determining the position and orientation of the end effector given the joint values, for the inverse kinematics the joints values are known and the problem is to find the position and orientation of the end effector.

For a parallel mechanism, inverse kinematic solution is very straight forward and can be performed using basic geometric approaches, but forward kinematic solution is very difficult, because we could have multiple solutions due to the mechanism structure [1] . The two common approaches to solve the forward kinematic problem consist of a polynomial based approach and a numerical solution.

Some authors [2], [3], [4], [5], [6] have solved the forward kinematics problem using purely geometric analysis, which results in high degree polynomials with multiple solutions. Therefore, obtaining a unique solution is difficult due to the complex manipulation of mathematical equations that we have to perform.

On the other hand, some other authors [7], [8], [9], have established numeric methods to solve forward kinematics using for example: interval analysis or kinematics mapping method. However, the forward kinematics of a parallel mechanism is still a challenging task due to the complicated formulation procedures and high computational cost.

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In this paper the Stewart - Gough (SG) Platform is studied. The SG platform is the most known parallel mechanism and is used in many robotic fields. Some applications of the SG platform are: flight simulators, haptic devices, milling machines, underwater vehicles, micro mechanisms, medical instrumentation and even shaking tables (earthquake simulators, landing deck for helicopters [10]).

There are many variants of a general Stewart Gough Platform (according to arrangements of the connecting joints in the fixed and moving platform), the different types have been proposed in order to find a closed form solution and simplify the forward kinematics problem.

For a general type, the polynomial analysis results in 12 equations with 12 unknowns, where each equation is of second degree highly nonlinear. Since the equations are of second degree, the Bezout number is 2^{12}, resulting in 4096 solutions. However, taking some considerations, some researchers have found up to 40 real solutions [8].

Other common type of SG platform is known as 3-6 platform. In this case, some researchers [11], [12] have shown that the forward kinematics solution results in a 16th-order polynomial. The other approach to solve the forward kinematics is to use a numerical method, which has shown to be more suitable to solve the forward kinematic problem. Some researchers [11], [12] have proposed numerical procedures using the Newton - Raphson method. However, they are used three unknown angles related with the moving platform orientation resulting in some problems due to the calculation of the partial derivative matrix and its inverse.

The aim of this paper is to present a new methodology that simplifies the forward kinematic problem, resulting in a highly efficient formulation approach. The proposed solution consists in modeling the constraints of movement for all moving parts, especially the joints, performing what is called a multibody formulation.

The idea of a multibody formulation is basically to built for each joint that links the body of the robot, a set of equations that define the constraints on their movement so that the joint variables are included in these equations [13]. Once the constraint function is defined, we may use a numerical method to find the kinematic solution. In order to avoid the problem reported with the orientation (Euler's Angles) of the moving platform, a generalized coordinates vector using quaternions is applied.

This article is organized as follows. First, the geometry of

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the mechanism is presented. Then, a numerical method for solving the forward kinematics based on a restriction function is detailed and Newton-Raphson iterative method is applied. Finally, a simulator to verify the results obtained is presented, MATLAB® as programming language and a platform testing is used.

II. The Stewart-Gough Platform

The Stewart-Gough platform consists of a fixed base and a moving platform which are joined together by six identical limbs. Each limb connect the moving platform to the fixed base using passive spherical joints (S). Also, these limbs are compose of two links, an upper member and a lower member, connected by a prismatic joint ($\underline{P}$), as shown in Fig. 1.

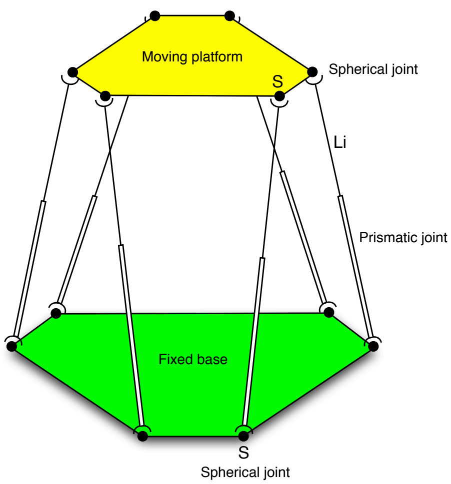

Fig. 1. Stewart-Gough platform.

Hydraulics or linear actuators can be used to change the length of the limbs and thus, to control de location (position and orientation) of the moving platform. In addition, the lower spherical joints can be change to universal joints (U) without losing the overall degrees of freedom of the mechanism, resulting in a 6U$\underline{P}$S robot.

A. Degrees of Freedom of the Mechanism

The Degrees of Freedom (DoF) of a mechanism are the number of independent parameters to specify the configuration of the mechamism [3]. According to Grübler and Kutzbatch, the DoF of a mechanism are given by:

$$F = \lambda(n - j - 1) + \sum_{i=1}^{n} f_i \tag{1}$$

where,

F : DoF of the mechanism.
λ : DoF of the space in which the robot will work.
n : number of links in the mechanism, including the base.
j : number of joints in the mechanism.
f_i : degrees of relative motion permitted in the joint i.

For example, considering a Stewart Gough platform (6U$\underline{P}$S) we have, $\lambda = 6, n = 14, j_1 = 6, j_2 = 6, j_3 = 6$. Substituting this values into (1), we obtain

$$F = 6(14 - 18 - 1) + (6 \times 1 + 6 \times 2 + 6 \times 3) = 6 \text{ DoF}$$

B. Geometry of the Mechanism

The geometry of the mechanism can be found taking some geometric considerations. To begin the analysis consider Fig. 2, where two Cartesian coordinate systems are attached to the fixed base and moving platform, respectively. Hence, the relative orientation between the moving reference frame P_{uvw} and the fixed reference frame O_{xyx} is given by the rotation matrix ${}^0R_P$, a vector-loop equation for the ith leg can be written as:

$${}^O\overrightarrow{a}_i + \overrightarrow{L}_i = \overrightarrow{p} + {}^O R_P \; {}^P\overrightarrow{b}_i \tag{2}$$

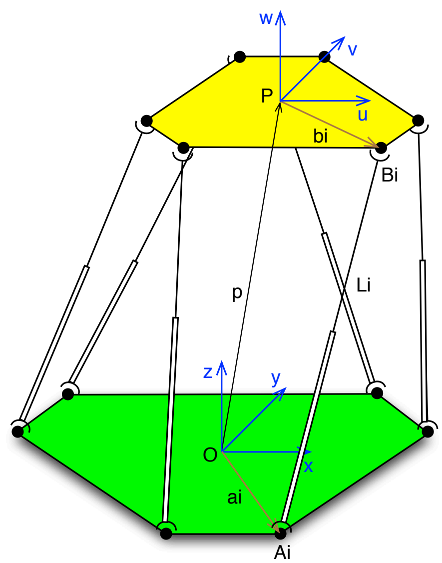

Fig. 2. Geometry of a Stewart-Gough platform.

where ${}^P\overrightarrow{b}_i$ is the vector localization of the anchoring of the ith limb on the moving platform expressed on the moving reference frame P_{uvw} , ${}^O\overrightarrow{a}_i$ is the vector localization of the anchoring of the ith limb on the fixed base expressed on the fixed reference frame O_{uvw} and $\overrightarrow{p}$ is the position vector of the centroid in the moving platform.

Thus, the length of each leg is given by:

$$\overrightarrow{L}_i = \| \overrightarrow{p} + {}^0 R_P \; {}^P\overrightarrow{b}_i - \overrightarrow{a_i} \| \tag{3}$$

To completely describe the location of the moving platform with respect to the fixed base we can write six times (3), one for each leg ($i = 1, 2, \ldots, 6$).

III. Forward Kinematic Analysis

The problem of the forward kinematic solution is to find the vector position $\overrightarrow{p}$ and the orientation of the moving platform expressed by the rotation matrix ${}^0R_P$, given the limb lengths L_i for($i = 1, 2, \ldots, 6$).

The position vector has three scalar unknowns, while the rotation matrix contains nine scalar unknowns. According to Tsai [3], and taking some considerations we obtained 12 equations in 12 unknowns.

The method proposed in this paper consists in the formulation of a multibody model of restrictions and then applying the Newton-Raphson method to approximate the solution. This is an iterative numerical method which starts with a initial estimated value of the the vector position and the orientation of the moving platform.

To apply the method lets consider Fig. 3, where two positions for the end-effector have been taken, first one in the "home" position and the other one in an arbitrary position.

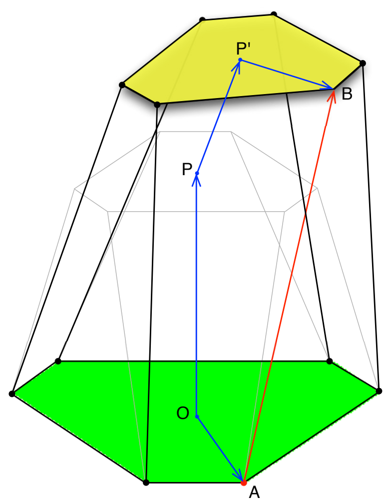

Fig. 3. Configuration Chosen for the forward kinematics Solution.

The analysis begins estimating the position of the end-effector and writing the estimation as a generalized coordinates vector, that is:

$$q = \begin{bmatrix} p & r \end{bmatrix} \tag{4}$$

Where p is the estimated position of the moving platform (x_0, y_0, z_0), and r is an Hamiltonian Quaternion (e_0, e_1, e_2, e_3) which describes the estimated orientation of the moving platform. To find the Quaternion, we begin estimating the Euler's angles (ψ, θ, ϕ) of the moving platform and then calculating its equivalent Quaternion.

Then, a loop-closure equation can be written for the ith limb as:

$$\overrightarrow{OP} + \overrightarrow{PP'} + \overrightarrow{P'B} - \overrightarrow{OA} - \overrightarrow{AB} = 0 \tag{5}$$

Thus, from (5), an objective function for the first limb can be written as:

$$\phi(q) = l(q) - l_0 \tag{6}$$

Where:

$l(q) = ||\overrightarrow{OP}|| + ||\overrightarrow{PP'}|| + ||\overrightarrow{P'B}|| - ||\overrightarrow{OA}||$, and

$l_0 = ||\overrightarrow{AB}||$

Vector $\overrightarrow{OP}$, represents the end-effector position at "home" (where,$\psi = 0, \theta = 0, \phi = 0$), $\overrightarrow{PP'}$ is the displacement vector from the center of the moving platform at "home" to the center of the estimated position of the moving platform, $\overrightarrow{P'B}$ represents the distance from joint position attached to the moving platform to the center of the platform, $\overrightarrow{OA}$ is the actuator position from the origin (centroid of the fixed base), $\overrightarrow{AB}$ is the length of the link that connected the fixed base with the moving platform.

Following the same procedure for the other five kinematic chains, the constraint function for the entire system can be written as:

$$F(q) = \begin{bmatrix} \phi_1(q) \\ \phi_2(q) \\ \phi_3(q) \\ \phi_4(q) \\ \phi_5(q) \\ \phi_6(q) \end{bmatrix} = \begin{bmatrix} l_1(q) - l_{0,1} \\ l_2(q) - l_{0,2} \\ l_3(q) - l_{0,3} \\ l_4(q) - l_{0,4} \\ l_5(q) - l_{0,5} \\ l_6(q) - l_{0,6} \end{bmatrix} \tag{7}$$

In order to find the real position of the moving platform, we can start making a Taylor series expansion of the restrictions function. If we take the first two terms in the series and neglect the higher orders terms we have:

$$F(q) = F_q(q)\Delta q \tag{8}$$

In (8), $F_q(q)$, represents the restrictions function derivate and is called the restrictions function Jacobian, the Jacobian is given by [13]:

$$F_q = \begin{bmatrix} U_1^T & -2U_1^T R\tilde{a}_1' G \\ U_2^T & -2U_2^T R\tilde{a}_2' G \\ U_3^T & -2U_3^T R\tilde{a}_3' G \\ U_4^T & -2U_4^T R\tilde{a}_4' G \\ U_5^T & -2U_5^T R\tilde{a}_5' G \\ U_6^T & -2U_6^T R\tilde{a}_6' G \end{bmatrix} \tag{9}$$

Where:

U_i : Unit vector of the restrictions function.

R : Rotation Matrix of the end-effector.

$\tilde{a}_i$: Skew matrix related with the joint position of the fixed base.

G : Rotation matrix that links the moving platform with the variation of the Quaternion.

As defined in [13], the matrix G in (9) can be expressed using Euler's parameters as:

$$G = \begin{bmatrix} -e_1 & e_0 & e_3 & -e_2 \\ -e_2 & -e_3 & e_0 & e_1 \\ -e_3 & e_2 & -e_1 & e_0 \end{bmatrix} \tag{10}$$

Once F_q is known, Eq.(8) can be written as:

$$\Delta q = -F(q)F_q^{\dagger}(q) \tag{11}$$

Equation (11) is known as Newton-Rhapson Method, where $F_q^{\dagger}$ is a pseudoinverse matrix. Then, the estimated value is updated iteratively via :

$$q_i = q_{i-1} + \Delta q \tag{12}$$

The algorithm continues and the forward kinematic solution is obtained when $\|F(q)\| \leq \varepsilon$, where ε is a fixed threshold. Fig. 4, shows a flowchart for the forward kinematic solution.

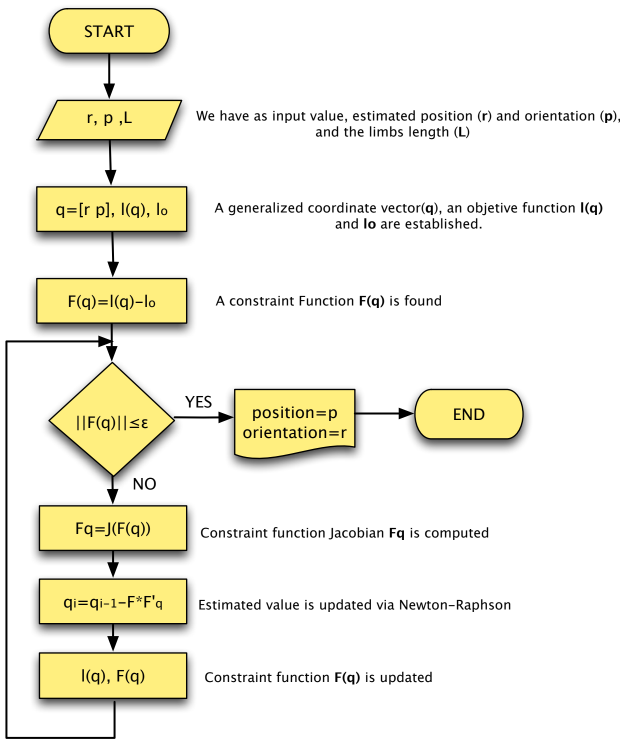

Fig. 4. Forward kinematic solution algorithm [14].

IV. Implementation

The tests have been performed using the software MATLAB® $R2014a$. To validate the forward kinematics solutions for the SG Platform proposed in this paper, a series of functions was developed. The main function has as input value the geometry of the robot (Links, fixed base and moving platform dimensions), the lengths of the limbs and the estimated position and orientation (Euler's Angles: roll,pitch and yaw) of the moving platform.

The program consists of 7 functions, function structure is shown in Fig 5., while a screenshot of the program running is depicted in Fig. 6.

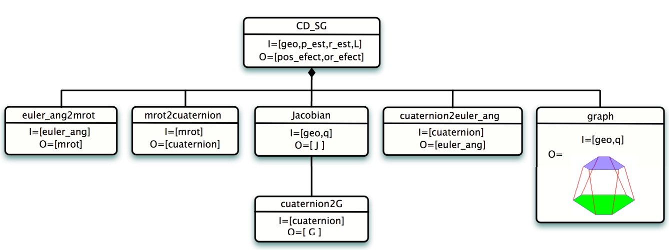

Fig. 5. Simulator Functions.

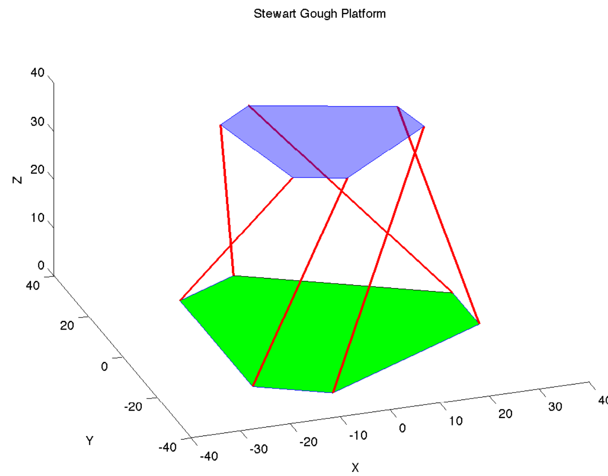

Fig. 6. Stewart Gough Platform Simulation.

A. Tests Performed

For the Tests performed we considered that the coordinates of the A_i points were:

$$A_1[28.9778 \quad 7.7646 \quad 0]; \quad A_2[-7.7646 \quad 28.9778 \quad 0]$$

$$A_3[-21.2132 \quad 21.2132 \quad 0]; \quad A_4[-21.2132 \quad -21.2132 \quad 0]$$

$$A_5[-7.7646 \quad -28.9778 \quad 0]; \quad A_6[28.9778 \quad -7.7646 \quad 0]$$

while the coordinates of the B_i at home, respect to the fixed base (point O) were:

$$B_1[14.1421 \quad 14.1421 \quad 40]; \quad B_2[5.1764 \quad 19.3185 \quad 40]$$

$$B_3[-19.3185 \quad 5.1764 \quad 40]; \quad B_4[-19.3185 \quad -5.1731 \quad 40]$$

$$B_5[5.1764 \quad -19.3185 \quad 40]; \quad B_6[14.1421 \quad -14.1421 \quad 40]$$

Furthermore, the position of the centroid of the moving platform at home is: P[0 0 40] and the orientation is: $roll = picth = yaw = 0°$.

For the tests, two sets of the leg lengths were taken. Then, for each set, 5 random estimated values of the position and orientation of the moving platform were considered, and the real values were found.

The computer used to perform the tests, has the following features:

- Brand: *MacBook Pro*
- Processor: *2.3 GHz Intel Core i5*
- RAM: *16 GB*
- Storage: *1 TB (SSHD)*

Test 1

In test 1, the position of the leg lengths were:

$$L_1 = 55.8558 \quad L_2 = 62.5313 \quad L_3 = 52.7436$$

$$L_4 = 55.1457 \quad L_5 = 44.7972 \quad L_6 = 51.9910$$

while the real values of the position and orientation for these values are:

$$p = [0 \;\; 0 \;\; 50] \quad \text{and} \quad roll = 20°; pitch = 0°, yaw = -30°$$

Table I shows the five estimated random values of the position and orientation and its computation time. For the five estimated values, the algorithm converged quickly and the real values were found.

TABLE I

Est. position (x,y,z)	Est. Orientation (ψ, θ, ϕ)	Computation Time (sec)
$[0, 20, 20]$	$[10, 100, 5]$	0.02227
$[0, 30, 60]$	$[0, -20, -10]$	0.01247
$[20, -15, 70]$	$[20, -20, 50]$	0.02547
$[-20, 5, 50]$	$[-20, -20, -50]$	0.02532
$[20, -10, 40]$	$[60, 70, 50]$	0.01196

Fig. 7 shows the real configuration and the five positions and orientations that were estimated.

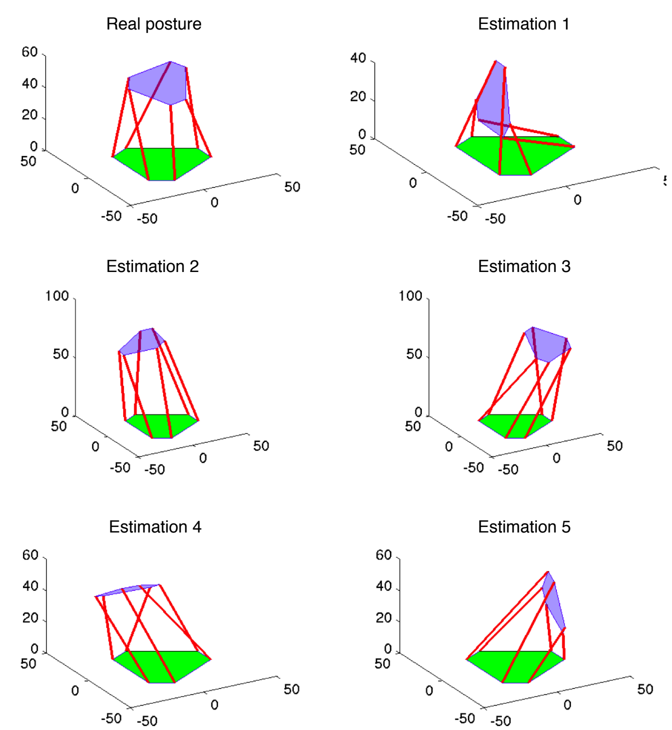

Fig. 7. Postures for Test 1. The real one and the estimations.

Test 2

In test 2, the position of the leg lengths were:

$$L_1 = 45.9508 \quad L_2 = 45.5433 \quad L_3 = 47.5475$$

$$L_4 = 49.2052 \quad L_5 = 51.0617 \quad L_6 = 36.3669$$

while the real values of the position and orientation for these values are:

$$p = [10 \;\; 10 \;\; 40] \quad \text{and} \quad roll = 10°; pitch = 10°, yaw = 20°$$

Table II shows the five estimated random values of the position and orientation for test 2, and its computation time. In all the cases the real values were obtained.

TABLE II

Est. position (x,y,z)	Est. Orientation (ψ, θ, ϕ)	Computation Time (sec)
$[10, -20, 30]$	$[0, 20, -10]$	0.005378
$[50, -20, 60]$	$[0, -20, 50]$	0.01111
$[-20, 30, 70]$	$[40, 50, 50]$	0.02328
$[0, -20, 30]$	$[-10, -20, -30]$	0.06947
$[40, 0, 70]$	$[0, 0, 0]$	0.01149

Fig. 8 shows the real configuration and the five positions and orientations that were estimated.

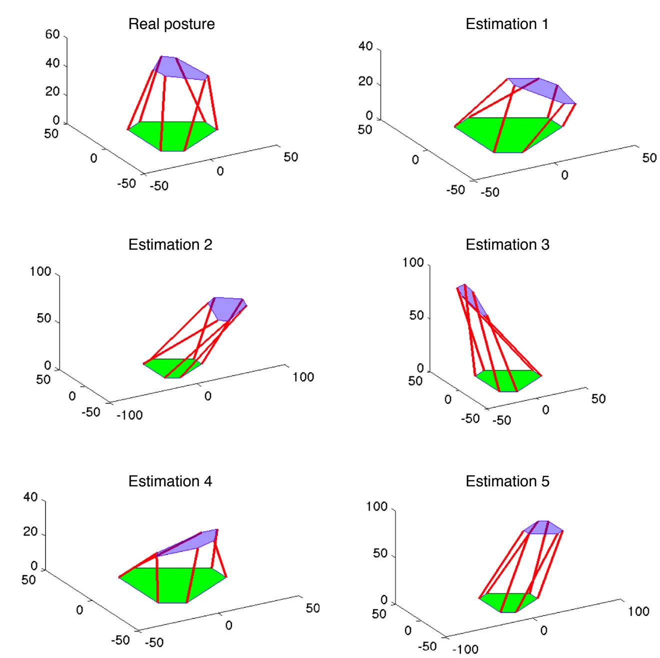

Fig. 8. Postures for Test 2. The real one and the initial estimations.

V. Conclusions and Discussions

In this paper a new methodology to deal with the forward kinematic of a general Stewart-Gough platform was presented. The Analysis consists in a multibody formulation and a constraint function. Then, an estimated position and orientation is iteratively corrected with the Newton - Rhapson numerical method, obtaining a quick and unique solution for the forward kinematics problem.

In general, from the results obtained, we can conclude that the method presented provides an interesting alternative for numerically solving the forward kinematics of a parallel robot.

Furthermore, to validate the methodology proposed in this paper, a design of a simulator using MATLAB was presented.

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